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Operability-Based Determination of Feasible Control Constraints for Several High-Dimensional Nonsquare Industrial Processes

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High-dimensional nonsquare systems, with more outputs than inputs, are common in industrial chemical processes. For such systems, it is impossible to control all the outputs at specific set-points. Interval control is needed for, at least, some of the output variables. An operability-based methodology, formulated in a Linear Programming framework, systematically determines the feasible set of the steady-state output constraints of high-dimensional nonsquare linear Model Predictive Controllers. These controllers are related to several industrial-scale chemical processes provided by Air Products and Chemicals and DuPont. It is shown that, for the operable cases, the constrained region of operation can be reduced, without causing infeasibilities, by a factor of 10^3–10^7 for systems that have an output dimensionality of 6–15. For the inoperable examples, the amount of constraint relaxation necessary to make the control problem feasible at the steady-state is also calculated. *AIChE J,* 56: 1249–1261, 2010
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Introduction and Problem Definition

Model predictive control (MPC) is a long standing multivariable constrained control methodology that utilizes an explicit process model to predict the future behavior of a chemical plant. At each control interval, the MPC algorithm attempts to optimize the future plant behavior by computing a sequence of future manipulated variable adjustments. The first of the optimal sequence of calculated input moves is implemented into the plant and the entire calculation is repeated at subsequent control intervals using updated process measurements.[1–4] This optimal control sequence is calculated by minimizing the error between the predicted future outputs and their specified reference trajectories over a horizon, subject to process constraints on the inputs and outputs. Output weights are used to represent the relative importance of each output variable according to the process control objectives, which

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can be based on economic, environmental, or safety factors. MPC has been extensively studied in academia and widely accepted in the chemical industry for its ability to handle complex multivariable and highly interactive process control problems.[1] However, most academic studies involving MPC are related to square systems. MPC-type controllers in industrial practice aim to control nonsquare systems in which there are more controlled outputs than manipulated inputs. In such systems, it is impossible to control all the outputs at specific set-points because there are fewer degrees of freedom available than the controlled variables (CVs). Apart from the common nonsquare nature of some chemical processes, a system with more outputs than inputs may also occur if one of the actuators of an original square system is operating at saturated levels (Lima FV and Georgakis C, submitted). Only a few references explicitly address the use of advanced control algorithms in the control of constrained nonsquare systems.[1,4,5–7]

On the basis of the input constraints, generally specified a priori due to the physical limitations of the process, an important design task is to determine the steady-state output ranges within which one wants to control the process. The improper selection of these constraints can make the controller infeasible when a disturbance moves the process far away from its usual operating region. Past practice requires that output constraints are enforced whenever feasible and softened whenever they become infeasible.[4,6,8,9] The premise of this approach is that the output constraints represent desired ranges of operation that can be violated if necessary. However, some economic and safety constraints are critical and cannot be softened. Moreover, if the output constraints are relaxed more than necessary, less tight control may result for some of the variables. This causes the operating point of the process to stray further away from the true economic optimum, which is often at the boundary of the acceptable region of operation. A successful controller in industrial applications must maintain the system as close to the constraints as possible without violating them.[1] Thus, one must determine the amount of softening, without having to resort to the complicated task of simulating all possible cases.[10]

The operability methodology originally introduced for square systems[11] and extended for nonsquare systems[12] provides a method for selecting such output constraints systematically, so that they are as tight as possible but also do not render the controller infeasible (see Refs. 13–18 for other process operability related approaches). Specifically for nonsquare systems, the *interval operability* framework was introduced[12] to assess the input–output open-loop operability of multivariable nonsquare systems at the steady-state, a necessary condition for the overall process operability. Simply stated, the operability framework quantifies the ability of a process to change from one steady-state to another and reject expected disturbances, utilizing the available set of inputs. The application of this framework to high-dimensional square and nonsquare systems is discussed in another publication (Lima FV and Georgakis C, submitted), where a linear programming (LP)-based approach is introduced to calculate the tightest feasible set of steady-state output constraints when interval operability is necessary.

On the basis of these nonsquare operability concepts (Lima FV and Georgakis C, submitted),[12] we examine the application of the developed LP-based methodology to the determination of output constraints at the steady-state of several industrial high-dimensional nonsquare linear MPC controllers. This adds credibility to the proposed approach for challenging industrial processes. It also enables verification of the achievability of control objectives before implementing the MPC controller. The approach calculates the tightest possible steady-state output constraints, enabling the computation of the hyper-volume reduction of the constrained region for initially operable systems, or the necessary constraint relaxation to make the control problem feasible for initially inoperable systems. Although we focus here on the determination of the steady-state constraints, the calculation of more relaxed output constraints during transient can be facilitated once steady-state constraints become available. Moreover, the steady-state output constraints calculated here are also applicable during transient for overdamped or critically damped systems, for cases when the input dynamics are faster than the disturbance dynamics. For the opposite case, when disturbance dynamics are faster than the input ones, and for underdamped systems in general, where overshoots may occur during process operation, dynamic operability analysis should be performed to calculate the amount of constraint relaxation necessary in order to prevent the occurrence of transient infeasibilities.

Finally, the proposed framework uses the steady-state version of the process model that the MPC controller will use, and can be employed offline at the design stage before the controller is deployed. It may also be employed online, making real-time adaptation of the control objectives possible, depending on the current state of the process. The main contribution of this manuscript is the study of high-dimensional process examples. This is motivated by the fact that industrial process control is characterized by large, multivariable, and constrained problems.[9]

Motivating Example

To motivate the determination of output constraints for MPC controllers, we first consider the steam methane reformer (SMR) process example provided by Air Products and Chemicals.[19] This process has nine CVs, four manipulated variables (MVs), and one disturbance variable (DV), which characterizes it as a high-dimensional nonsquare system. For demonstration purposes, the system was slightly modified from the original one, which will be presented in the Results section along with the process description. All simulations in this section are performed using DMCplus™ (dynamic matrix control; AspenTech), an established and industry-proven multivariable-constrained controller.[1,20] The SMR step response model obtained from DMCplus is shown in Figure 1, where y, u, and d represent outputs, inputs, and DVs, respectively. The original limits for all these variables are shown in Table 1. A tighter, but still feasible set of steady-state output constraints for all the values of the disturbance is presented in Table 2. The importance of determining the steady-state output constraints will be demonstrated here using two cases, where the feasible subset of constraints for y_6 and y_7 are tightened one at a time and all the other output limits in Table 2 are kept constant. In both cases, all simulations start with the process operating at the steady-state conditions given in Table 1.

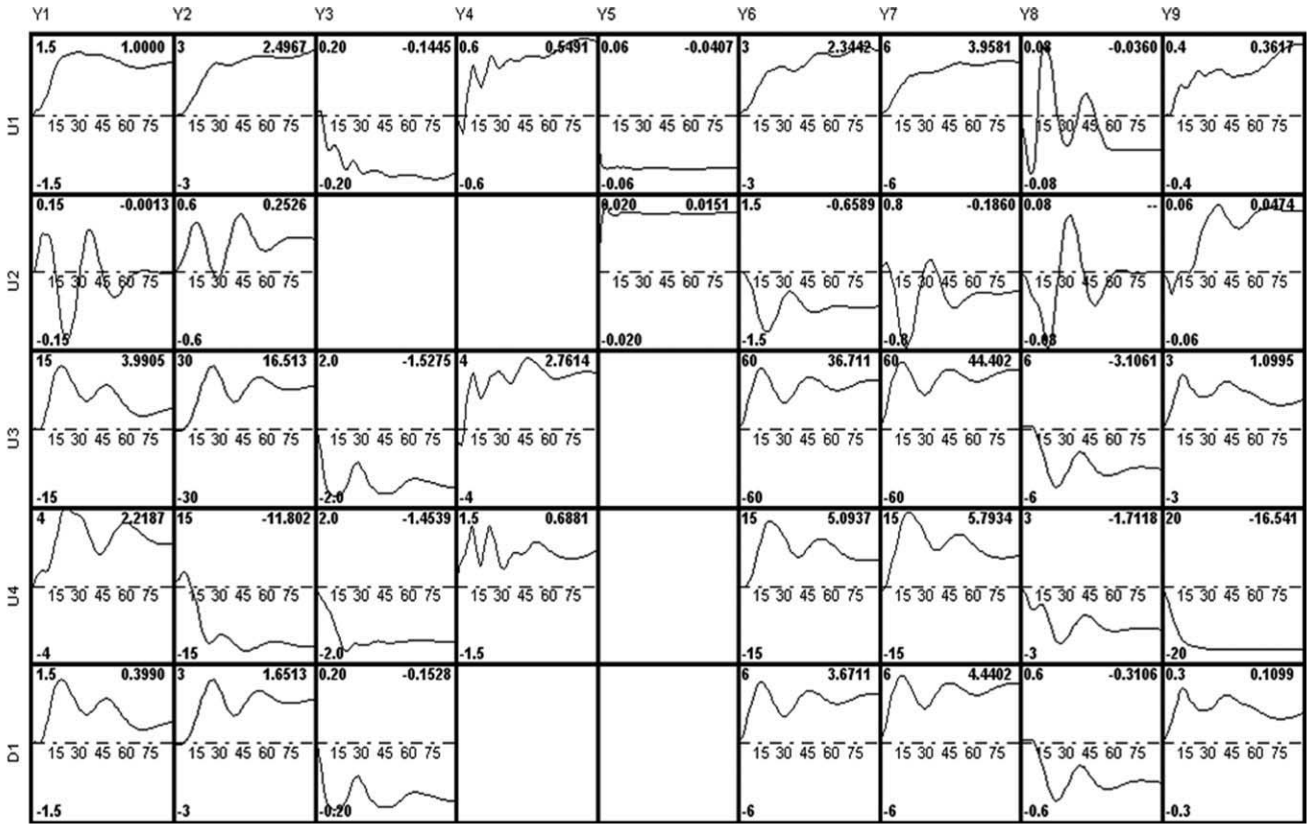

Figure 1. Step response model for the SMR problem.

Case 1: Tightening the y_6 constraints by 1 unit at each end

In this case, the limits for y_6 and y_7 are specified in Table 3. Assume that the process is operating at its steady-state and a disturbance is inserted with its maximum value ($d_1 = 10$). The time dependence for y_6 and y_7 as they approach a new steady-state are shown in Figures 2 and 3, respectively. It is noticed that the new steady-state values for both outputs are within the steady-state constraints given in Table 3. Moreover, constraint violations during transient operation, as occurs for y_6, are acceptable here because we are addressing only steady-state operability. However, as observed in Figure 4, the new y_5 steady-state value is clearly outside of its constraints, which characterizes an infeasible control problem for this set of output constraints. No steady-state infeasibilities were present in the other output variables.

Case 2: Tightening the y_7 constraints by 10 units at each end

Here, the output constraints specified in Table 4 are used. Starting the simulation again from the original steady-state and inserting the disturbance at its maximum value as previously, the trends for y_6, y_7, and y_5 until they reach a new steady-state are shown in Figures 5, 6, and 7 respectively. Even though the limits for y_7 were reduced in this case by 20 units in total, these figures show that the control problem is still feasible because the new steady-state values for all these outputs are within their limits. No infeasibilities occurred for the other output variables in this case.

Table 1. Original Limits for the SMR Process Variables

Process Variable	Low Limit	High Limit	Steady-State
u_1	10.00	48.00	29.00
u_2	60.00	140.00	100.00
u_3	0.20	2.00	1.10
u_4	−2.40	−0.70	−1.55
y_1	43.00	45.70	44.35
y_2	26.90	161.30	94.10
y_3	0.80	2.20	1.50
y_4	0.00	43.00	21.50
y_5	1.70	1.90	1.80
y_6	424.70	438.20	431.45
y_7	430.10	591.40	510.75
y_8	3.20	7.50	5.35
y_9	21.50	52.70	37.10
d_1	−10.00	10.00	0.00

Table 2. Set of Feasible Steady-State Limits for the SMR Output Variables (tighter than Limits in Table 1)

CV	Feasible Low Limit	Feasible High Limit
y_1	44.00	44.70
y_2	77.30	110.90
y_3	1.32	1.68
y_4	16.12	26.88
y_5	1.77	1.83
y_6	429.76	433.14
y_7	490.58	530.92
y_8	4.81	5.89
y_9	33.20	41.00

Table 3. Tightening y_6 Constraints: y_6 and y_7 Limits

	Original		New	
CV	Low Limit	High Limit	Low Limit	High Limit
y_6	429.76	433.14	430.76	432.14
y_7	490.58	530.92	490.58	530.92

Furthermore, the y_7 constraints could still be further reduced if desired and the problem would remain feasible. Therefore, there is a clear need for a methodology that determines the output bounds to identify the minimum range between the maximum and minimum constraints that prevent infeasibilities in the MPC controller in the presence of disturbances.

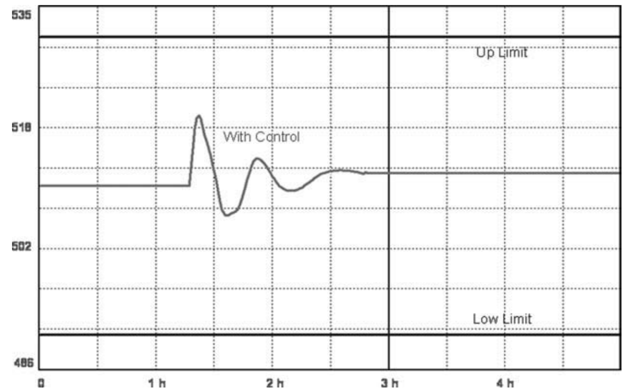

Figure 3. Time dependence of y_7 for case 1.

Interval Operability of Multivariable Nonsquare Systems

To quantify the steady-state operability and to determine the output constraints of nonsquare linear systems, process outputs are classified into two categories: *set-point controlled*: variables that are controlled at exact set-point values (production rates and product qualities) and *set-interval controlled*: variables that are controlled within specified ranges (pressure, temperature, and level). In the latter case, we refer to the operability as *interval operability*.[12] The set-point and range variables are selected according to the process control objectives. When assessing the interval operability of a process, one aims to fix critical outputs (e.g., variables representing economics or safety) to be controlled tightly over narrow ranges or even at set-points, allowing the others to vary within maximum and minimum limits. Process outputs must have at least one feasible operating point within the desired interval. To clarify the concept of interval operability, it is first necessary to define some useful sets. The Available Input Set (AIS) is the set of values that the process input variables can take, based on the input constraints of the process. For an $n \times m \times q$ (outputs $\times$ inputs $\times$ disturbances) nonsquare system, the AIS is given by:

$$\mathrm{AIS} = \{\mathbf{u} | u_i^{\min} \le u_i \le u_i^{\max}; 1 \le i \le m\}$$

Examples of these variables are valves or actuators that can be manipulated during the process operation. The desired output set (DOS) is given by the ranges of the outputs that are desired to be achieved and might be represented by:

$$\mathrm{DOS} = \{\mathbf{y} | y_i^{\min} \le y_i \le y_i^{\max}; 1 \le i \le n\}$$

These output variables could represent quantities such as product qualities and production rate, as well as reactor temperatures, levels, and conversions, among others. The DOS should be based on product specifications, market demand, safety considerations, equipment, and machinery limits and emission regulations. Finally, the expected disturbance set (EDS) represents the expected steady-state values of the disturbances:

$$\mathrm{EDS} = \{\mathbf{d} | d_i^{\min} \le d_i \le d_i^{\max}; 1 \le i \le q\}$$

Examples of these steady-state disturbances are process uncertainties, including parameters established during the design stage, such as heat transfer coefficients, reaction rate constants, and physical properties; or actual operating parameters, such as quality and flow rates of feed streams, catalyst activity, heat exchanger fouling, ambient temperature fluctuations, and plant-model mismatch.

On the basis of the steady-state model of the process, expressed by the process gain matrix (**G**) and the disturbance gain matrix ($\mathbf{G}_\mathrm{d}$), the achievable output set for a specific

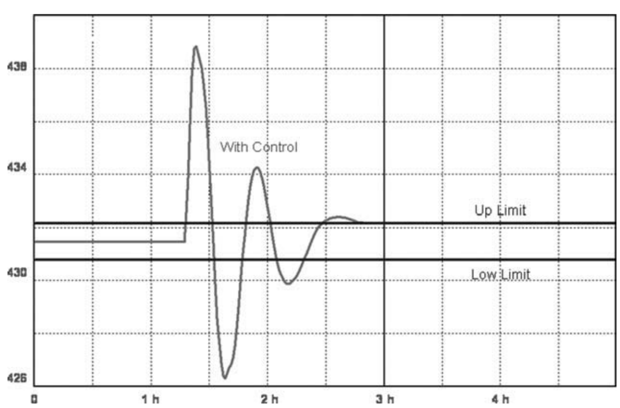

Figure 2. Time dependence of y_6 for case 1.

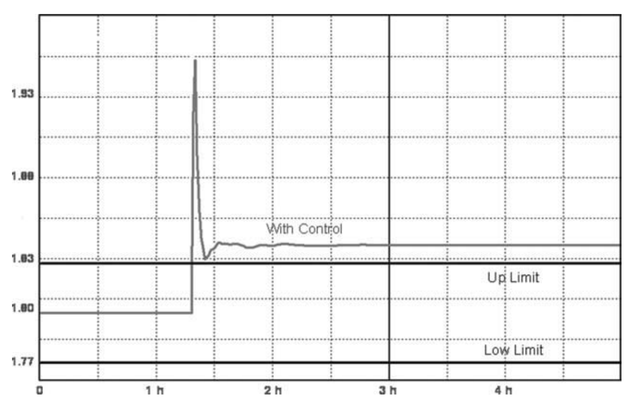

Figure 4. Time dependence of y_5 for case 1.

Table 4. Tightening y_7 Constraints: y_6 and y_7 Limits

	Original		New	
CV	Low Limit	High Limit	Low Limit	High Limit
y_6	429.76	433.14	429.76	433.14
y_7	490.58	530.92	500.58	520.92

disturbance vector (AOS(**d**)) is defined by the ranges of the outputs that can be achieved using the inputs inside the AIS:

$$\mathrm{AOS}(\mathbf{d}) = \{\mathbf{y} | \mathbf{y} = \mathbf{G}\mathbf{u} + \mathbf{G}_{\mathrm{d}}\mathbf{d}; \mathbf{u} \in \mathrm{AIS}\} \tag{1}$$

Thus, the AOS($\mathbf{d} = 0$) is a subset of an m-dimensional manifold in $\Re^n$. Varying the process disturbance vector, which can take values within the EDS (q-dimensional region), the AOS($\mathbf{d} = 0$) is shifted in the $\Re^n$ space along the directions determined by $\mathbf{G}_{\mathrm{d}}$ and by the **d** vector of the active disturbance values (Eq. 1). The values of **d** at the vertices of the EDS, representing the extreme cases, are of particular interest. The union of all shifted locations for all the possible disturbance values within the EDS yields the interval (subscript I) AOS ($\mathrm{AOS_I}$), which is a subset of $\Re^n$:

$$\mathrm{AOS_I} = \bigcup_{\mathbf{d} \in \mathbf{EDS}} \mathrm{AOS}(\mathbf{d})$$

To calculate the feasible output ranges, we will use the definition of the achievable output interval set (AOIS) given by Lima and Georgakis.[12,21,22] The AOIS was defined as the tightest possible feasible set of output constraints that can be achieved at the steady-state with the available ranges of the MVs when the disturbances remain within their expected values. It represents the minimum requirement for the constraints of each output to yield an operable system for all disturbance values, including the worst case. In other words, as long as the AOIS is a subset of the DOS (AOIS $\subseteq$ DOS) the system is interval operable for all cases. Thus, for high-dimensional and operable systems, the following hyper-volume ratio (HVR) quantifies how much the constrained region of operation can be reduced without rendering infeasibilities to the control problem[12]:

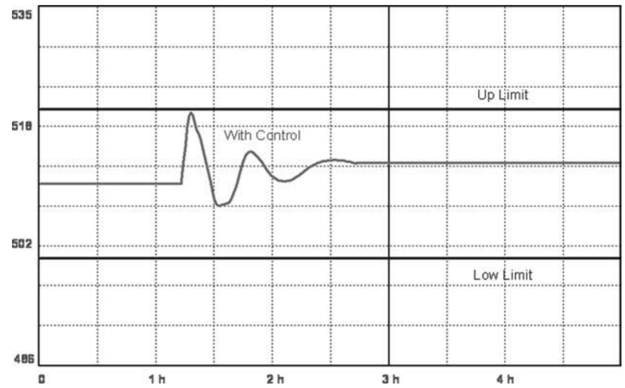

Figure 6. Time dependence of y_7 for case 2.

$$\mathrm{HVR} = \frac{\mu(\mathrm{DOS})}{\mu(\mathrm{AOIS})} \tag{2}$$

where μ represents a measure of the size of the set, which is usually the hyper-volume. Also, this ratio is higher than or equal to 1 for operable systems. If the above-mentioned interval operability condition is not satisfied, infeasibilities will occur for some of the disturbance values. In that case, the AOIS provides the exact amount of constraint relaxation for each one of the outputs to cope with these infeasibilities.

To perform the AOIS calculation, an iterative approach was first presented in Lima and Georgakis.[12] However, this methodology has some limitations that make its online implementation difficult (Lima FV and Georgakis C, submitted). Here, a LP-based approach, detailed in another publication (Lima FV and Georgakis C, submitted), will be applied to perform high-dimensional AOIS calculations in $\Re^n$ for any nonsquare linear system at steady-state, to be used in the determination of MPC output constraints. The algorithm of the previously developed framework is adapted here for the purpose of its online implementation. The proposed approach may address any input–output linear system with n outputs, m inputs, and q disturbances. Thus, the AIS, DOS, and EDS are subsets of $\Re^m$, $\Re^n$, and $\Re^q$, respectively. This approach is presented in later section, where all the high-dimensional computational geometry calculations are performed using the multiparametric toolbox (MPT) in MATLAB (Mathworks™, Inc).[23]

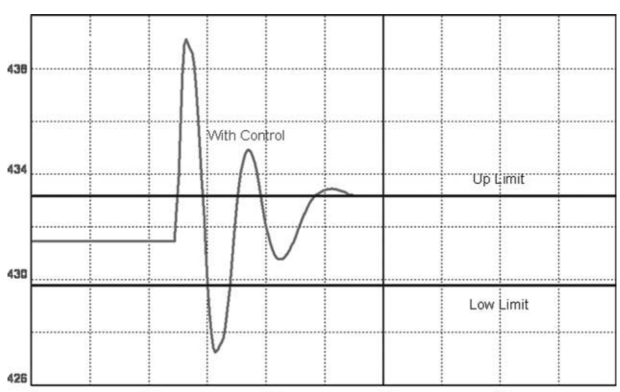

Figure 5. Time dependence of y_6 for case 2.

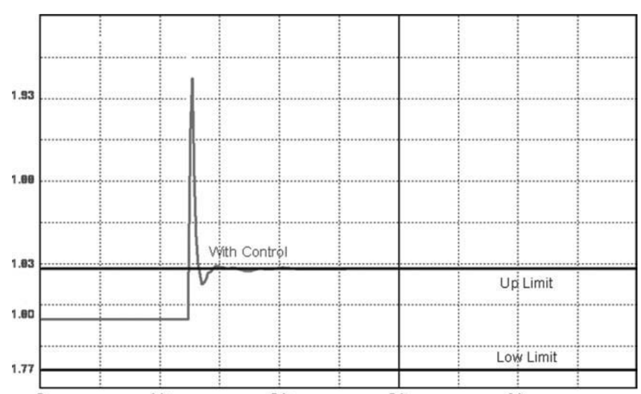

Figure 7. Time dependence of y_5 for case 2.

Calculation of AOIS: LP approach

Two sets of output parameters are considered in the AOIS calculation: the steady-state target point ($\mathbf{y}_0$) and the relative output weights ($\mathbf{w}$). The relative output weights represent the tightness with which each output will be controlled around its desired target and will affect the aspect ratio of the corresponding sides of the calculated AOIS. For example, an aspect ratio of 1:10 between two outputs assures that one will be controlled 10 times more tightly, approximating set-point control. Several examples of AOIS calculations using different weights and output targets have been previously presented.[12]

The set of points that characterize the vertices of the AOS can be easily calculated by directly mapping the vertices of the AIS and EDS using the linear steady-state process model (Eq. 1). The calculation of AOIS in $\Re^n$ is performed by formulating the interval operability problem in a LP framework, where the AOS and the AOIS polytopes are described as a system of inequalities in the LP formulation. An overview of the algorithm for this calculation, presented in Lima FV and Georgakis C, submitted, is the following:

(1) Define the relative weights $w_1, w_2, \ldots w_n$ that quantify the tightness within which each output needs to be controlled;

(2) Select one of the extreme disturbance vectors $\mathbf{d} = \mathbf{d}_i$, $i = 1, 2, \ldots, k$, which corresponds to one of the $k = 2^q$ vertices of EDS. Calculate AOS($\mathbf{d}_i$) (Eq. 1) and the corresponding linear equalities and inequalities that define this set (see details later);

(3) Define a family of n-dimensional orthogonal parallelepipeds, $P(\alpha)$, self-similar among them, centered at the target value of the outputs ($\mathbf{y}_0$), where the scalar α affects their sizes by the following set of inequalities:

$$P(\alpha) = \{\mathbf{y} | -\mathbf{b} \leq \mathbf{y} - \mathbf{y_0} \leq \mathbf{b}\} \tag{3}$$

where

$$\mathbf{b} = \left(\frac{\alpha}{w_1}, \frac{\alpha}{w_2}, \ldots, \frac{\alpha}{w_n}\right)^T;$$

$$\mathbf{y_0} = (y_{01}, y_{02}, \ldots, y_{0n})^T;$$

and

$$\mathbf{y} = (y_1, y_2, \ldots, y_n)^T.$$

(4) Calculate the minimum value of α, α_i, such that $P(\alpha_i)$ and AOS($\mathbf{d}_i$) have a single common point $\mathbf{v}_i$, by solving the LP problem below:

$$\begin{aligned} &\alpha_i = \min_{\mathbf{x}} \alpha = \min_{\mathbf{x}} \mathbf{f^T x}, \\ &\text{where } \mathbf{f} = [0\,0 \ldots 0\,1]^T; \text{and } \mathbf{x} = [y_1 y_2 \cdots y_n \ \alpha]^T = [\mathbf{y}^T \alpha]^T; \\ &\text{while } \mathbf{y} = \mathbf{v}_i \in P(\alpha_i) \cap \text{AOS}(\mathbf{d}_i) \end{aligned} \tag{4}$$

(5) Repeat steps 2–4 above for a total of $k = 2^q$ times to calculate the set of k points: $\mathbf{v}_1, \mathbf{v}_2, \ldots \mathbf{v}_k$;

(6) The final AOIS is the smallest orthogonal parallelepiped in $\Re^n$ that includes all the k $\mathbf{v}_i$ points from the LP solutions (AOIS $= OP(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \ldots \mathbf{v}_k)$). This set defines the tightest set of output constraints that makes the process operable for the output target $\mathbf{y}_0$ and all the disturbance values inside the EDS.

Details on Step 2 above:

The $2m$ inequalities of AOS($\mathbf{d}_i$) are calculated from Eq. (1) as follows:

$$\mathbf{u}_{\min} \leq \mathbf{G}^{\dagger}(\mathbf{y} - \mathbf{G}_{\mathrm{d}}\mathbf{d}_i) \leq \mathbf{u}_{\max} \tag{5}$$

where $\mathbf{G}^{\dagger} = (\mathbf{G}^T\,\mathbf{G})^{-1}\,\mathbf{G}^T$ is the pseudo-inverse of $\mathbf{G}$.[24] The $(n - m)$ equality constraints that also contribute to the definition of AOS($\mathbf{d}_i$) are as follows:

$$\mathbf{U}_2^T\mathbf{y} = \mathbf{U}_2^T\mathbf{G}_{\mathrm{d}}\mathbf{d}_i \tag{6}$$

where $\mathbf{U}_2$ is an $n \times (n - m)$ matrix whose columns correspond to the left singular vectors associated with the zero singular values of $\mathbf{G}$.

Remark. The method presented here is applicable to any process described by a linear steady-state model between the input (disturbance and control) and output variables, where the input-constrained sets are defined by the orthogonal parallelepipeds (EDS, AIS). Thus far, we have examined several high-dimensional systems and we have not noticed any computational limitations of the proposed approach. It is certain that such limitations will arise for higher-dimensional systems, but we do not know at present for what dimensionality that would occur. We expect though that the most important limitations of this method are its linear and steady-state character, rather than the size of the problem that it can address within a reasonable computational time. Also, the parallelepiped nature of the constrained sets examined here can be extended to any other convex polytopic constrained set, described by linear inequality constraints.

Results: Industrial Examples

To demonstrate the effectiveness of the proposed LP approach, high-dimensional systems characterizing industrial problems are addressed here. Specifically, the feasible sets of output constraints are calculated for the following process examples: (1) SMR provided by Air Products and Chemicals; (2) dryer control problem (DCP), and (3) sheet forming control problem (SFCP), both provided by DuPont.

Steam methane reformer

Process Description. SMR reforms natural gas (NG) by combining it with steam to generate a synthesis gas which is purified in a pressure swing absorption (PSA) unit to yield a high purity hydrogen product. The objective is to efficiently produce the hydrogen product on demand. This product is compressed and supplied to multiple consumers via a pipeline, while the purge gas from the PSA is returned to the reformer as fuel. Steam is also produced for consumption as a by-product of the process and sold via pipeline.[19] A simplified flow diagram of this process is shown in Figure 8. The process inputs for this example are the NG feed, steam, NG fuel, and combustion air (not shown in the diagram). The returning purge gas from the PSA is the primary source of disturbances and the process outputs include the hydrogen product, steam product, as well as multiple constraining process temperatures, and there are several additional

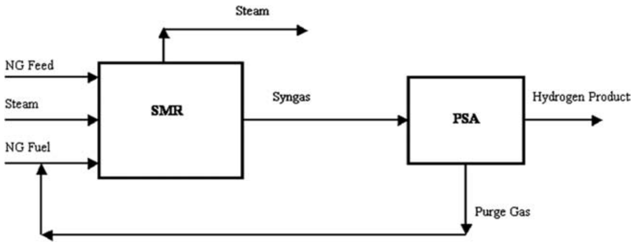

Figure 8. Simplified flow diagram of the SMR process example.

constraints. Altogether, this example SMR process has nine outputs, four inputs, and one disturbance. The steady-state gain process model and input sets that describe this system are the following:

$$\begin{pmatrix} \delta y_1 \\ \delta y_2 \\ \delta y_3 \\ \delta y_4 \\ \delta y_5 \\ \delta y_6 \\ \delta y_7 \\ \delta y_8 \\ \delta y_9 \end{pmatrix} = \begin{pmatrix} 1.00 & 0 & 3.99 & 2.22 \\ 2.50 & 0.25 & 16.51 & -11.80 \\ -0.14 & 0 & -1.53 & -1.45 \\ 0.55 & 0 & 2.76 & 0.69 \\ -0.04 & 0.02 & 0 & 0 \\ 2.34 & -0.66 & 36.71 & 5.09 \\ 3.96 & -0.19 & 44.40 & 5.79 \\ -0.04 & 0 & -3.11 & -1.71 \\ 0.36 & 0.05 & 1.10 & -16.54 \end{pmatrix} \begin{pmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \\ \delta u_4 \end{pmatrix} + \begin{pmatrix} 0.40 \\ 1.65 \\ -0.15 \\ 0 \\ 0 \\ 3.67 \\ 4.44 \\ -0.31 \\ 0.11 \end{pmatrix} \delta d_1 \qquad (7)$$

$$\text{AIS} = \{\mathbf{u} \in \Re^4 | 10 \le u_1 \le 48; 60 \le u_2 \le 140; 0.2 \le u_3 \le 2.0; -2.4 \le u_4 \le -0.7\}$$
$$\text{EDS} = \{d_1 | -4 \le d_1 \le 4\}$$
$$\text{with } \delta\mathbf{y} = \mathbf{y} - \mathbf{y}_{ss}; \delta\mathbf{u} = \mathbf{u} - \mathbf{u}_{ss}; \delta d_1 = d_1 - d_{1,ss}$$

where $\delta\mathbf{y}$, $\delta\mathbf{u}$, and δd_1 are deviation variables from the steady-state values for the outputs ($\mathbf{y}_{ss}$), inputs ($\mathbf{u}_{ss}$), and disturbance ($d_{1,ss}$), respectively. The original output constraints (DOS) are given in Table 1. The relative output weights are $\mathbf{w} = (0.10, 0.20, 0.04, 1.00, 0.04, 0.04, 0.04, 0.04, 0.04)^T$. The tightest feasible set of output constraints (AOIS) for the SMR problem is calculated next for two different cases, corresponding to different steady-state output targets.

Case 1. Consider the steady-state values for all the process variables presented in Table 1 and the following target for the outputs: $\mathbf{y}_0 = (44.35, 94.10, 1.50, 21.50, 1.80, 431.45, 510.75, 5.35, 37.1)^T$. The calculated AOIS ranges that will enable a constrained controller to operate without infeasibilities around the corresponding target are presented in Table 5. The constraint reduction factor (CRF) for each output i (CRF_i), where $i = 1, 2, \ldots, n$, are also shown in the same table. These reduction factors are calculated using Eq. 8.[12]

$$\text{CRF}_i = \frac{(y_i^u - y_i^l)}{(y_i^{du} - y_i^{dl})} \qquad (8)$$

As observed in Table 5, the output constraints were successfully reduced for this scenario by a factor that ranges approximately between 1.43 and 36.44. Notice that, as expected, the reduction for y_4 is significantly higher than the others due to its higher relative importance, expressed by its weight defined earlier. To quantify the total reduction of the constrained region, we use the previously defined HVR, as the ratio between the hyper-volumes of the original constrained region and the calculated constrained region, which can be obtained by the following equation:

$$\text{HVR} = \prod_{i=1}^{n} \text{CRF}_i \qquad (9)$$

Using the CRF values in Table 5 and the equation above, we calculated a HVR of 9.15×10^3 times for this case. This implies that the new 9-D space within which the process can be effectively operated is 9.15×10^3 times smaller than the original constrained space. These calculated new limits were validated by running DMCplus simulations for the extreme values of the disturbances. To conclude the analysis for this case, the average computational time of 10 repeated AOIS calculations was only 0.14 s (Dell PC with a 3.0-GHz Intel

Table 5. Results for SMR Example—Case 1: Original and Determined Set of Output Constraints and Constraint Reduction Factors for Each of the Process Outputs

	Original		Determined		
Process Outputs	Lower Bound (y^l)	Upper Bound (y^u)	Lower Bound (y^{dl})	Upper Bound (y^{du})	CRF
y_1	43.00	45.70	43.98	44.72	3.65
y_2	26.90	161.30	84.89	103.31	7.30
y_3	0.80	2.20	1.02	1.98	1.46
y_4	0	43.00	20.91	22.09	36.44
y_5	1.70	1.90	1.73	1.87	1.43
y_6	424.70	438.20	426.82	436.08	1.46
y_7	430.10	591.40	455.48	566.02	1.46
y_8	3.20	7.50	3.87	6.83	1.45
y_9	21.50	52.70	26.41	47.79	1.46

Table 6. Results for SMR Example—Case 2: Original and Determined set of Output Constraints for Each of the Process Outputs

Process Outputs	Original Lower Bound (y^l)	Original Upper Bound (y^u)	Determined Lower Bound (y^{dl})	Determined Upper Bound (y^{du})
y_1	43.00	45.70	45.33	46.07
y_2	26.90	161.30	121.55	139.97
y_3	0.80	2.20	0.32	1.28
y_4	0	43.00	31.61	32.79
y_5	1.70	1.90	1.63	1.77
y_6	424.70	438.20	420.07	429.33
y_7	430.10	591.40	494.65	605.13
y_8	3.20	7.50	5.58	8.56
y_9	21.50	52.70	35.02	56.40

Pentium 4 processor), which can be easily calculated online during an MPC controller application.

Case 2. Consider a different steady-state operating point characterized by the following input and output values: $\mathbf{u}_{ss} = (42.03,\ 111.14,\ 0.54,\ -1.90)^T$ and $\mathbf{y}_{ss} = (45.70,\ 130.76,\ 0.80,\ 32.20,\ 1.70,\ 424.70,\ 549.89,\ 7.07,\ 45.71)^T$. It is desired to keep the outputs at or near the steady-state, thus $\mathbf{y}_0 = \mathbf{y}_{ss}$. In this case, the target for outputs y_1, y_3, y_5, and y_6 are at their respective initial constraints. Thus, to guarantee a feasible operation around this $\mathbf{y}_0$, in the presence of the disturbance within the EDS, the output constraints corresponding to at least these four outputs must be relaxed. The minimum amount of relaxation for each output constraint is calculated here and it is shown in Table 6. In particular, the upper limit for y_1 should be moved to 46.07 and the lower limits for y_3, y_5, and y_6 should be moved to 0.32, 1.63, and 420.07, respectively, assuming that they would not violate any hard constraints of the process.

Dryer Control Problem

Process Description. The DCP consists of a low pressure particle drying application (see process Flowsheet in Figure 9). Ambient air, heated by NG combustion, is mixed with a flow of wet particles. The mixture obtained is conveyed through a dryer/scrubber/baghouse unit operation by an induction fan. The process is highly interactive and subject to internal flow and pressure disturbances due to caking of partially dried material on the sides of the dryer. This is measured by the Mixer Amps DV. There is also a significant measurement noise present on y_4. The measurement noises associated with the other outputs are neglected for this analysis. The process objective is to put as much material as possible through the unit subject to constraints on measurements characterizing temperatures, pressures, and final particle moisture. Also, the minimal consumption of NG is desired to the extent possible. This process has six CVs, four MVs, and two DVs that are presented in Table 7. Also, it is described by the following system of equations, sets, and relative output weights:

$$\begin{pmatrix} \delta y_1 \\ \delta y_2 \\ \delta y_3 \\ \delta y_4 \\ \delta y_5 \\ \delta y_6 \end{pmatrix} = \begin{pmatrix} 9.00 & -5.10 & -0.80 & 0.31 \\ 0.06 & -0.05 & 0.03 & 0 \\ 0.70 & -0.40 & 0 & 0 \\ -44.00 & -3.00 & 3.50 & 1.56 \\ 0 & 9.60 & 0 & 0 \\ 0.60 & 0 & -0.03 & -0.13 \end{pmatrix} \begin{pmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \\ \delta u_4 \end{pmatrix} + \begin{pmatrix} 0.62 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1.10 & 1 \\ -1.50 & 0 \\ 0.04 & 0 \end{pmatrix} \begin{pmatrix} \delta d_1 \\ \delta d_2 \end{pmatrix} \tag{10}$$

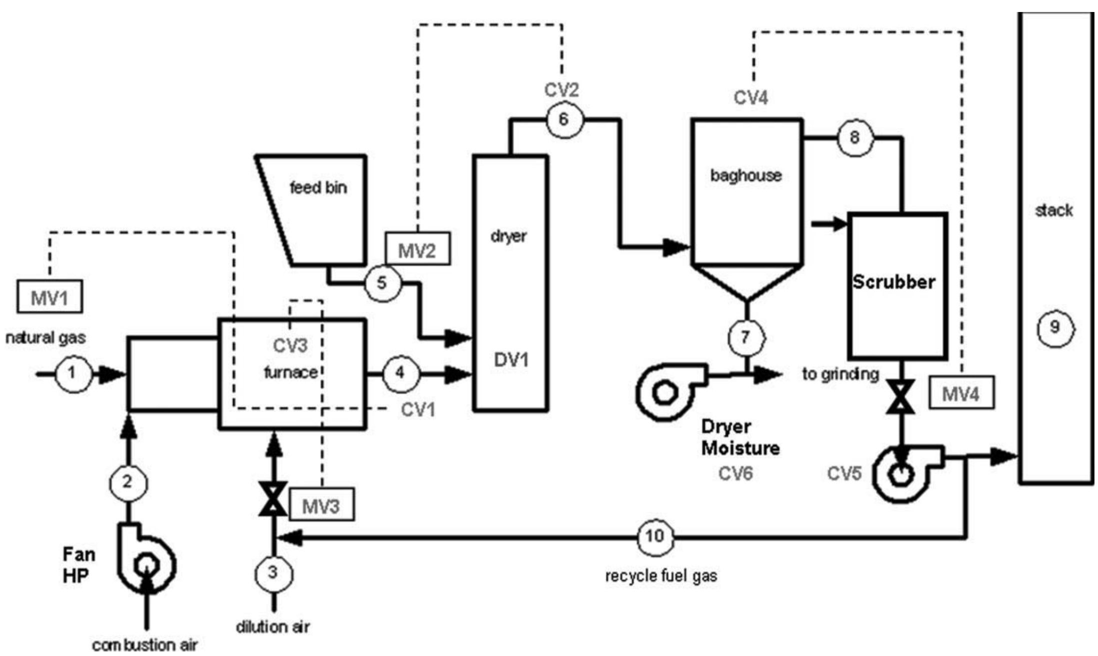

Figure 9. DCP process flowsheet.

Table 7. DCP Process Variables

Process Variable	Description
MV1 (u_1)	Gas Flow SP
MV2 (u_2)	Feed Screw Amps SP
MV3 (u_3)	Secondary Damper VP
MV4 (u_4)	Scrubber Orifice VP
CV1 (y_1)	Inlet Temperature
CV2 (y_2)	Outlet Temperature
CV3 (y_3)	Combustion Chamber Pressure
CV4 (y_4)	Bag-house Pressure
CV5 (y_5)	Main Fan HP
CV6 (y_6)	Predicted Exit Moisture
DV1 (d_1)	Mixer Amps
DV2 (d_2)	Inlet Moisture

$$\mathrm{AIS} = \{\mathbf{u} \in \Re^4 | 30 \le u_1 \le 95; 40 \le u_2 \le 95; 0 \le u_3 \le 100; 20 \le u_4 \le 90\}$$

$$\mathrm{EDS} = \{\mathbf{d} \in \Re^2 | 30 \le d_1 \le 70; -30 \le d_2 \le 30\}$$

$$\mathbf{w} = \left(\frac{1}{3}, \frac{1}{4}, \frac{1}{4}, 1, \frac{1}{4}, \frac{1}{2}\right)^T$$

This problem is slightly more general than the one presented in Lima and Georgakis[12] by accounting here for the effect that a second disturbance has on y_4. The original upper and lower sets of output constraints are shown in Table 8. It is assumed that the steady-state is at the midpoint of each of the process variable ranges and the output target is $\mathbf{y}_0 = (950, -2, -25, 135, 1475, 0.5)^T$. The AOIS, represented by the calculated ranges around $\mathbf{y}_0$, and the CRFs for each of the outputs in this case are also presented in Table 8. Thus, the hyper-volume of the original 6-D space of operation around the initial steady-state could be significantly reduced (HVR = 5.96×10^6) without affecting the feasibility of the process.

In the following cases, we will move the output target from its initial value to another one to accomplish the following desirable process control objectives in order of priority:

(1) Control y_4 as close as possible to its lower limit (100);

(2) control y_6 as close as possible to its upper limit (1);

(3) control the other outputs around their desired operating points (y_1, y_2, y_3, y_5) = (965, −2, −30, 1500) if possible, with **w** dictating their relative importance;

Case 1: Changing the Output Target to y_0 = (960, −2, −25, 100, 1475, 1)^T

In this case, the target values for y_4 and y_6 are specified at their lower and upper constraints, respectively. The calculated ranges (AOIS) for this scenario are shown in Table 9. Notice that the determined constraints for y_4 and y_6 violate their steady-state limits at the lower and upper bounds, respectively. The calculated amount of back-off[18] from the target point necessary to guarantee that constraint violations at the steady-state will not occur in the presence of the disturbances is also shown in Table 9. Adding this amount to y_4 and y_6 target values, the output target will now be moved to the back-off point, $\mathbf{y}_0$ = (960, −2, −25, 101.12, 1475, 0.97)^T, which is inside the feasible operating region. The calculated AOIS ranges when the back-off point is the target are shown in Table 10. For this target, the system can operate without violating any steady-state feasibility constraints in the presence of disturbances. Also, now y_4 and y_6 can be controlled very closely to their lower and upper limits, respectively, accomplishing the two most important control objectives. Furthermore, the calculated operating ranges for y_2 include its desired operating point ($y_2 = -2$), though this is not true for y_1 ($y_1 = 965$), y_3 ($y_3 = -30$), and y_5 ($y_5 = 1500$). In Case 2, we will try to accomplish this for y_1, the most important of these three outputs.

Table 8. Dryer Control Problem Results: Original and Determined Bounds, Target Point, and Constraint Reduction Factors for Each of the Controlled Variables

	Original			Determined		
CV	Lower Bound (y^l)	Upper Bound (y^u)	Target Point (y_0)	Lower Bound (y^{dl})	Upper Bound (y^{du})	CRF
y_1	900.00	1000.00	950.00	945.96	954.04	12.38
y_2	−4.00	0.00	−2.00	−2.22	−1.78	9.09
y_3	−40.00	−10.00	−25.00	−26.62	−23.38	9.26
y_4	100.00	170.00	135.00	134.05	135.95	36.84
y_5	1300.00	1650.00	1475.00	1456.20	1493.80	9.31
y_6	0.00	1.00	0.50	0.47	0.53	16.67

Table 9. Dryer Control Problem Results—Case 1: Original and Determined Bounds, Target Point and Calculated Steady-State Back-Off for Each of the Controlled Variables

	Original			Determined		
CV	Lower Bound (y^l)	Upper Bound (y^u)	Target Point (y_0)	Lower Bound (y^{dl})	Upper Bound (y^{du})	Steady-State Back-Off
y_1	900.00	1000.00	960.00	955.21	962.93	–
y_2	−4.00	0.00	−2.00	−2.19	−1.74	–
y_3	-40.00	−10.00	−25.00	−26.64	−23.36	–
y_4	100.00	170.00	100.00	98.88	100.80	1.12
y_5	1300.00	1650.00	1475.00	1452.70	1490.90	–
y_6	0.00	1.00	1.00	0.97	1.03	−0.03

Table 10. Dryer Control Problem Results Using the Back-Off Point as the Target: Original and Determined Bounds, Desired Operating Point, and Constraint Reduction Factor for Each of the Controlled Variables

CV	Original		Back-Off Point (y_0)	Determined		CRF
	Lower Bound (y^l)	Upper Bound (y^u)		Lower Bound (y^{dl})	Upper Bound (y^{du})	
y_1	900.00	1000.00	960.00	955.18	962.85	13.04
y_2	−4.00	0.00	−2.00	−2.18	−1.74	9.09
y_3	−40.00	−10.00	−25.00	−26.64	−23.36	9.15
y_4	100.00	170.00	101.12	100.00	101.91	36.65
y_5	1300.00	1650.00	1475.00	1452.50	1490.80	9.14
y_6	0.00	1.00	0.97	0.94	1.00	16.67

Case 2: Changing the Relative Weight of y_1

The relative output weights (**w**) represent the tightness with which each output will be controlled around its target point. Here, the calculated intervals for y_1 will be relaxed by reducing its weight from 1/3 to 1/5. This enables the inclusion of its desired operating point ($y_1 = 965$) within its calculated ranges. Table 11 shows the calculated AOIS ranges and the CRFs in this case for the same target point (back-off) as in Case 1. Notice that the most important control objectives are still satisfied in this scenario.

Case 3: Increasing y_4 and y_6 Weights: Set-Point Control Problem

In order to control the two most important output variables (y_4 and y_6) at their set-points, their weights (w_4 and w_6, respectively) must be increased. Here, it is assumed that the y_4 and y_6 set-points are at their lower ($y_{0,4} = 100$) and upper limits ($y_{0,6} = 1$), respectively, and their weights are increased from (w_4, w_6) = (1, 1/2) to (100, 50). For the other variables, the original output weights in Eq. 10, (w_1, w_2, w_3, w_5) = (1/3, 1/4, 1/4, 1/4) and the output target used in case 1, ($y_{0,1}$, $y_{0,2}$, $y_{0,3}$, $y_{0,5}$) = (960, −2, −25, 1475), are assumed. The calculated AOIS ranges and CRFs for these specified conditions are shown in Table 12. Notice that y_4 and y_6 could be practically operated at their set-points, if these were the only control objectives to be achieved.

Finally, the typical computational time of the AOIS calculations for all cases was 0.33 s. Also, the determined constraints for each case were validated at the steady-state, as in the SMR example presented earlier, by running DMCplus simulations.

Sheet forming control problem

Process Description. The objective of the SFCP is to control the sheet thickness of the scanning *B* gauge at 15 different points as uniformly as possible around different targets. Thus, there are 15 CVs, which correspond to the thicknesses in the cross-direction, with the same relative weight. Moreover, this process has nine MVs and one DV. Figure 10 shows a schematic representation of this process and Table 13 presents the process variables description. The steady-state gain model, the sets within which the input and output variables are constrained and the relative weights of the output variables are given by the following equations:

$$\begin{pmatrix} \delta y_1 \\ \delta y_2 \\ \delta y_3 \\ \delta y_4 \\ \delta y_5 \\ \delta y_6 \\ \delta y_7 \\ \delta y_8 \\ \delta y_9 \\ \delta y_{10} \\ \delta y_{11} \\ \delta y_{12} \\ \delta y_{13} \\ \delta y_{14} \\ \delta y_{15} \end{pmatrix} \begin{pmatrix} -2 & 0 & -1.8 & 0 & 0 & 0 & -0.00064 & 0 & 0 \\ -2 & 0 & -1.6 & 0 & 0 & 0 & -0.00035 & 0 & 0 \\ -2 & 0 & -1.4 & -2 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & -1.8 & 0 & 0 & 0 & -0.00036 & 0 \\ -2 & 0 & 0 & -1.6 & 0 & 0 & 0 & -0.00064 & 0 \\ -2 & 0 & 0 & 0 & -1.6 & 0 & 0 & -0.00084 & 0 \\ 0 & -2 & 0 & 0 & -1.8 & 0 & 0 & -0.00096 & 0 \\ 0 & -2 & 0 & 0 & -2 & 0 & 0 & -0.001 & 0 \\ 0 & -2 & 0 & 0 & -1.8 & 0 & 0 & -0.00096 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & -0.00084 & 0 \\ 0 & -2 & 0 & 0 & 0 & -1.8 & 0 & -0.00064 & 0 \\ 0 & -2 & 0 & 0 & 0 & -1.8 & 0 & -0.00036 & 0 \\ 0 & -2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & -1.8 & 0 & 0 & -0.00036 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & -0.00064 \end{pmatrix} \begin{pmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \\ \delta u_4 \\ \delta u_5 \\ \delta u_6 \\ \delta u_7 \\ \delta u_8 \\ \delta u_9 \end{pmatrix} \begin{pmatrix} 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \\ 1.7 \end{pmatrix} (\delta d_1)$$

$$\text{AIS} = \left\{ \begin{array}{c} u \in \Re^9 | 210 \le u_1 \le 230; 210 \le u_2 \le 230; 220 \le u_3 \le 235; \\ 220 \le u_4 \le 235; 220 \le u_5 \le 235; 220 \le u_6 \le 235; \\ 2000 \le u_7 \le 4000; 2000 \le u_8 \le 4000; 2000 \le u_9 \le 4000 \end{array} \right\}$$

Table 11. Dryer Control Problem Results—Case 2: Original and Determined Bounds, Target Point, and Constraint Reduction Factor for Each of the Controlled Variables

CV	Original		Target Point (y_0)	Determined		CRF
	Lower Bound (y^l)	Upper Bound (y^u)		Lower Bound (y^{dl})	Upper Bound (y^{du})	
y_1	900.00	1000.00	960.00	953.11	965.61	8.00
y_2	−4.00	0.00	−2.00	−2.18	−1.78	10.00
y_3	−40.00	−10.00	−25.00	−26.50	−23.50	10.00
y_4	100.00	170.00	101.12	100.15	101.91	39.77
y_5	1300.00	1650.00	1475.00	1455.70	1490.80	9.97
y_6	0.00	1.00	0.97	0.94	0.99	20.00

Table 12. Dryer Control Problem Results—Case 3: Original and Determined Bounds, Target Point, and Constraint Reduction Factor for Each of the Controlled Variables

CV	Original		Target Point (y_0)	Determined		CRF
	Lower Bound (y^l)	Upper Bound (y^u)		Lower Bound (y^{dl})	Upper Bound (y^{du})	
y_1	900.00	1000.00	960.00	955.09	963.21	12.32
y_2	−4.00	0.00	−2.00	−2.19	−1.73	8.70
y_3	−40.00	−10.00	−25.00	−26.67	−23.33	8.98
y_4	100.00	170.00	100.00	99.99	100.01	3598.60
y_5	1300.00	1650.00	1475.00	1452.10	1491.00	9.00
y_6	0.00	1.00	1.00	1.00	1.00	1799.30

$$\mathrm{EDS} = \{d \in \Re^1 | 130 \le d_1 \le 170\}; d_{ss} = 150; \quad \mathrm{DOS} = \{\mathbf{y} \in \Re^{15} | 1.9 \le y_i \le 2.3; 1 \le i \le 15\} \tag{11}$$

$$\mathbf{w} = (1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)^T$$

$$\mathbf{u}_{ss} = (220,220,227.5,227.5,227.5,227.5,3000,3000,3000)^T$$

$$\mathbf{y}_{ss} = (2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1)^T$$

The initial desired output constraints (DOS) are the same for all the outputs. The problem of determining the feasible set of output constraints will be addressed here by controlling the sheet thickness of specified zones, which are represented by the following set of zone variables ($\mathbf{z}$):

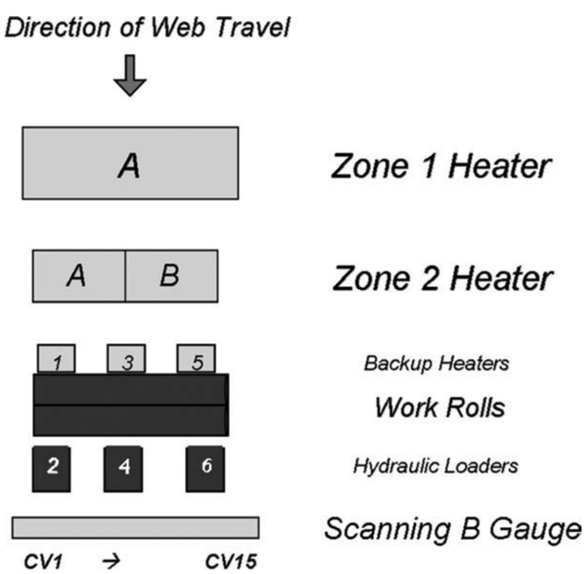

Figure 10. Schematic representation of the SFCP.

$$z_1 = \frac{y_1 + y_2}{2}; z_2 = y_3; z_3 = \frac{y_4 + y_5}{2}; z_4 = \frac{y_6 + y_7}{2}; z_5 = y_8; z_6 = \frac{y_9 + y_{10}}{2}; z_7 = \frac{y_{11} + y_{12}}{2}; z_8 = y_{13}; z_9 = \frac{y_{14} + y_{15}}{2};$$

where y_8 (z_5) corresponds to the measurement at the center of the sheet. The tightest feasible set of constraints (AOIS) will be calculated for these zone variables and then used for the outputs at the corresponding zone. This approach reduces the dimensionality of the original problem from 15×9 to a 9×9 square problem. The solution of this reduced problem provides an alternative approximate way to calculate the achievable constraints for the output variables using the properties of this distributed process. The results of two scenarios assuming different steady-state targets are presented next.

Case 1. Consider initially the problem of determining the tightest set of output constraints around the nominal target ($\mathbf{y}_0$) of 2.1 units for all the CVs. The AOIS ranges calculated for this case are:

$$\mathrm{AOIS} = \{\mathbf{z} \in \Re^9 | -4.65 \le z_i \le 8.85; 1 \le i \le 9, d_1 \in \mathrm{EDS}\}$$

Observe that, to have a feasible process operation for all the values of the disturbance, the zone constraints need to be relaxed to unrealistic negative values for their lower bounds.

Table 13. SFCP process variables

Process Variable	Description
u_1	Zone 1 Heater A Power SP
u_2	Zone 2 Heater A Power SP
u_3	Zone 2 Heater B Power SP
u_4–u_6	1, 3, 5 Backup Heater SPs
u_7–u_9	2, 4, 6 Hydraulic Loader SPs
y_1–y_{15}	Cross Machine Sheet Thicknesses at 15 Positions
d_1	Line Speed

This characterizes an inoperable system for some values of the disturbance. However, assuming a smaller range of disturbance values ($138 \le d_1 \le 162$) makes the system operable and very tight control can be achieved, which is indicated by the AOIS ranges calculated in this case:

$$\text{AOIS} = \{\mathbf{z} \in \Re^9 | 2.04 \le z_i \le 2.16; 1 \le i \le 9, 138 \le d_1 \le 162\}$$

The CRF for each zone and the ratio between the hypervolumes of the original constrained region and the determined constrained region (HVR) are:

$$\text{CRF} = 3.33$$
$$\text{HVR} = 5.08 \times 10^4$$

This implies that for the assumed disturbance values, the process could be operated feasibly within a constrained region 5.08×10^4 tighter than the region initially specified by the DOS. In fact, infeasibilities will not occur for a disturbance range as wide as $137.53 \le d_1 \le 162.47$, which is indicated by a calculated CRF value equal to 1 for all the zones. In this case, the original zone constraints are just enough to provide feasible operation for the specified disturbance range. Thus, if the disturbance range is slightly wider than this value, then infeasibilities will start occurring.

Case 2. Here, we will perform a similar analysis as earlier except that the output target ($\mathbf{y}_0$) is moved from 2.1 to 2.0 units. For this target, the system is again inoperable for the widest range of the disturbance. However, realistic AOIS ranges for the zone variables are obtained when the disturbance range is equal to $138 \le d_1 \le 162$. These ranges and the CRF for each zone are the following:

$$\text{AOIS} = \{\mathbf{z} \in \Re^9 | 1.92 \le z_i \le 2.08; 1 \le i \le 9, 138 \le d_1 \le 162\}$$
$$\text{CRF} = 2.50$$

Thus, for the specified conditions, the original constrained region could be again significantly reduced ($\text{HVR} = 3.81 \times 10^3$). In this case, if the disturbance range is gradually widened, the lower limit of the zone variables is reached for a range of $137.89 \le d_1 \le 162.11$, which is demonstrated by the calculated AOIS:

$$\text{AOIS} = \{\mathbf{z} \in \Re^9 | 1.90 \le z_i \le 2.10; 1 \le i \le 9, 137.89 \le d_1 \le 162.11\}$$

This limit is reached first due to the asymmetric position of the target within the ranges, which is closer to 1.9 than 2.3. For this scenario, even though the CRF value for each zone is 2, and thus $\text{HRV} = 2^9 = 512$, a slightly wider disturbance range would cause constraint violations at the lower end.

To conclude the analysis for this process example, the typical computational time for the AOIS calculations considering all cases was 0.19 seconds. Here as well, simulations were performed using DMCplus to validate the results for all cases. All these results correspond to the conservative set of constraints, representing the widest calculated thicknesses among the zones. Although tighter control would be possible for some zones, sheet-thickness uniformity is desirable, and the use of conservative limits does not affect process feasibility. If tighter control of the overall thickness is intended, design modifications to enlarge the AIS should be performed (Lima FV and Georgakis C, submitted).

Finally, using these calculated zones' results for the outputs within the corresponding zone, infeasibilities may occur for some outputs. This occurs when, for instance, one of the outputs within a zone demands more conservative control than its pair. The square approximation used here does not take this into account. Therefore, a simplification of a nonsquare system by a square one may provide a less realistic calculation of the AOIS (Lima FV and Georgakis C, submitted).[25]

Conclusions

We have used our recently presented operability-based methodology formulated in the LP framework to determine the feasible set of output constraints for nonsquare MPC controllers. We have addressed high-dimensional nonsquare systems, where some of the output variables need to be controlled within intervals rather than at set-points. Specifically, an algorithm to calculate the AOIS in $\Re^n$ was presented, where n is the number of outputs. This set represents the tightest possible operable set of output constraints that can be achieved, with the available range of the MVs and when the disturbances remain within their expected values. The proposed LP approach can accommodate different process operating points as well as different weights on the tightness of the desired control of each output, including the set-point control case. This approach enables operability calculations for high-dimensional problems in $\Re^n$ in fractions of a second. Additionally, we note that using the developed methodology, the exact amount of constraint reduction/relaxation for each of the outputs can be determined systematically and industrial processes can be examined without the need of trial and error simulations. Here this methodology was successfully applied to challenging industrial-scale examples provided by Air Products and Chemicals and DuPont. For the operable cases, a significant reduction of the constrained region, represented by the HVR between the initial and the calculated regions, was achieved characterized by values of HVR that ranged between 10^3 and 10^7.

Using the interval operability concept, MPC controllers can be designed during process operation to provide tight control of the outputs without the significant risk of making the controller operation infeasible. Our attention here was limited to operability calculations at the steady-state, a necessary condition for the overall process operability. The dynamic operability of nonsquare systems and its corresponding application to the design of MPC controllers is presently under investigation.

Acknowledgments

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Notation

$\mathbf{d}$ = disturbance variables
$\mathbf{d}_{ss}$ = steady-state for disturbance variables
$\mathbf{G}$ = process gain matrix
$\mathbf{G}_d$ = disturbance gain matrix
$\mathbf{G}^{\dagger}$ = pseudo-inverse of $\mathbf{G}$
m = number of input variables
n = number of output variables
$P(\alpha)$ = family of n-dimensional orthogonal parallelepipeds defined by α
q = number of disturbance variables
$\mathbf{u}$ = input or manipulated variables
$\mathbf{u}_{ss}$ = steady-state for input variables
U_2 = matrix associated with the left singular vectors of G
$\mathbf{v}_i$ = solution of LP problem i
$\mathbf{w}$ = relative output weights
$\mathbf{y}$ = output or controlled variables
$\mathbf{y}_0$ = output targets
$\mathbf{y}^{dl}$ = output determined lower bounds
$\mathbf{y}^{du}$ = output determined upper bounds
$\mathbf{y}^{l}$ = output original lower bounds
$\mathbf{y}^{u}$ = output original upper bounds
$\mathbf{y}_{ss}$ = steady-state for output variables
$\mathbf{z}$ = zone variables

Acronyms

AIS = available input set
AOIS = achievable output interval set
AOS = achievable output set
AOS_I = achievable output set for interval operability
CRF = constraint reduction factor
CV = controlled variable
DCP = dryer control problem
DMC = dynamic matrix control
DV = disturbance variable
DOS = desired output set
EDS = expected disturbance set
HP = horse power
HVR = hyper volume ratio
LP = linear programming
MPC = model predictive control
MPT = multiparametric toolbox
MV = manipulated variable
NG = natural gas
OP = orthogonal parallelepiped
PSA = pressure swing absorption
SFCP = sheet forming control problem
SMR = steam methane reformer
SP = set-point
VP = valve position

Greek letters

α = scalar that defines the size of orthogonal parallelepipeds $P(\alpha)$
$\delta\mathbf{d}$ = deviation variables from the steady-state for the disturbances
$\delta\mathbf{u}$ = deviation variables from the steady-state for the inputs
$\delta\mathbf{y}$ = deviation variables from the steady-state for the outputs
μ = measure of the size of a high-dimensional set

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